Seasonal Variation

Introduction

- An analysis of seasonal fluctuations over a period of years can also help in evaluating current sales.
- The typical sales of department stores, are expressed as indexes in Table 1.
- A typical set of monthly indexes consists of 12 indexes that are representative of the data for a 12-month period.

Table 1. Typical Seasonal Indexes for Department Store Sales

January	87.0	July	86.0
February	83.2	August	99.7
March	100.5	September	101.4
April	106.5	October	105.8
May	101.6	November	111.9
June	89.6	December	126.8

Determining a Seasonal Index

- Logically, there are four typical seasonal indexes for data reported quarterly.
- Each index is a percent, with the average for the year equal to 100.0; that is, each monthly index indicates the level of sales, production, or another variable in relation to the annual average of 100.0.
- A typical index of 87.0 for January indicates that sales (or whatever the variable is) are usually 13 percent below the average for the year.
- An index of 105.8 for October means that the variable is typically 5.8 percent above the annual average.

Determining a Seasonal Index

- Several methods have been developed to measure the typical seasonal fluctuation in a time series. The method most commonly used to compute the typical seasonal pattern is called the **ratio-to-moving-average method.**
- It eliminates the trend, cyclical, and irregular components from the original data (*Y*).
- In the following discussion, *T* refers to trend, *C* to cyclical, *S* to seasonal, and *I* to irregular variation. The numbers that result are called the *typical seasonal index.*
- We will discuss in detail the steps followed in arriving at typical seasonal indexes using the ratio-to-moving-average method. The data of interest might be monthly or quarterly.

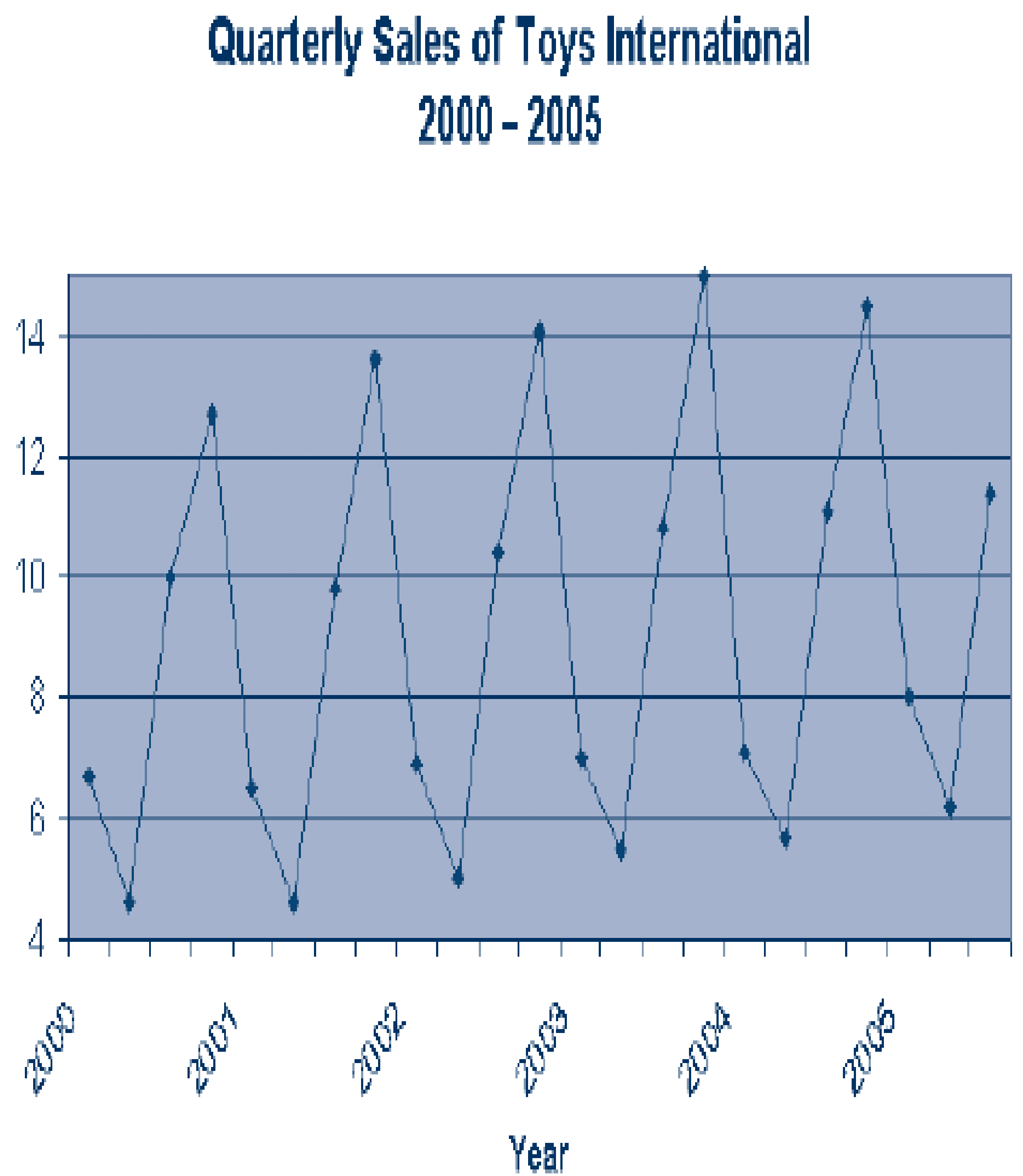
Example

- The following Table (Table 2) shows the quarterly sales for Toys International for the years 2000 through 2005.
- The sales are reported in millions of dollars. Determine a quarterly seasonal index using the ratio-to-moving-average method.

Table 2

Year	Winter	Spring	Summer	Fall
2000	6.7	4.6	10.0	12.7
2001	6.5	4.6	9.8	13.6
2002	6.9	5.0	10.4	14.1
2003	7.0	5.5	10.8	15.0
2004	7.1	5.7	11.1	14.5
2005	8.0	6.2	11.4	14.9

Year	Sales	Code
2000	6.7	1
	4.6	2
	10.0	3
	12.7	4
2001	6.5	5
	4.6	6
	9.8	7
	13.6	8
2002	6.9	9
	5.0	10
	10.4	11
	14.1	12
2003	7.0	13
	5.5	14
	10.8	15
	15.0	16
2004	7.1	17
	5.7	18
	11.1	19
	14.5	20
2005	8.0	21
	6.2	22
	11.4	23
	14.9	24



Example

- For each year, the fourth-quarter sales are the largest and the second-quarter sales are the smallest.
- Also, there is a moderate increase in the sales from one year to the next.
- Over the six-year period, the sales in the fourth quarter increased.
- If you connect these points in your mind, you can visualize fourth-quarter sales increasing for 2006.

There are six steps to determining the quarterly seasonal indexes.

Step 1 For the following discussion, the first step is to determine the four-quarter moving total for 2000.

- Starting with the winter quarter of 2000,

- we add \$6.7, \$4.6, \$10.0, and \$12.7. The total is \$34.0 (million). The four quarter total is “moved along” by adding the spring, summer, and fall sales of 2000 to the winter sales of 2001.
- The total is \$33.8 (million), found by 4.6+10.0+12.7+6.5.
- This procedure is continued for the quarterly sales for each of the six years.
- Column 2 of the following Table (Table 3) shows all of the moving totals.

Table 3

Year	Quarter	(1) Sales ($ millions)	(2) Four-Quarter Total	(3) Four-Quarter Moving Average	(4) Centred Moving Average	(5) Specific Seasonal
2000	Winter	6.7				
	Spring	4.6				
			34.0	8.500		
	Summer	10.0			8.475	1.180
			33.8	8.450		
	Fall	12.7			8.450	1.503
			33.8	8.450		
2001	Winter	6.5			8.425	0.772
			33.6	8.400		
	Spring	4.6			8.513	0.540
			34.5	8.625		
	Summer	9.8			8.675	1.130
			34.9	8.725		
	Fall	13.6			8.775	1.550
			35.3	8.825		
2002	Winter	6.9			8.900	0.775
			35.9	8.975		

(Continued)

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Year	Quarter	(1) Sales ($ millions)	(2) Four-Quarter Total	(3) Four-Quarter Moving Average	(4) Centred Moving Average	(5) Specific Seasonal
	Spring	5.0			9.038	0.553
			36.4	9.100		
	Summer	10.4			9.113	1.141
			36.5	9.125		
	Fall	14.1			9.188	1.535
			37.0	9.250		
2003	Winter	7.0			9.300	0.753
			37.4	9.350		
	Spring	5.5			9.463	0.581
			38.3	9.575		
	Summer	10.8			9.588	1.126
			38.4	9.600		
	Fall	15.0			9.625	1.558
			38.6	9.650		
2004	Winter	7.1			9.688	0.733
			38.9	9.725		
	Spring	5.7			9.663	0.590
			38.4	9.600		
	Summer	11.1			9.713	1.143
			39.3	9.825		
	Fall	14.5			9.888	1.466
			39.8	9.950		
2005	Winter	8.0			9.888	0.801
			40.1	10.025		
	Spring	6.2			10.075	0.615
			40.5	10.125		
	Summer	11.4				
	Fall	14.9				

- Note that the moving total 34.0 is positioned between the spring and summer sales of 2000. The next moving total, 33.8, is positioned between sales for summer and fall of 2000, and so on.
- Check the totals frequently to avoid arithmetic errors.

Step 2 Each quarterly moving total in column 2 is divided by 4 to give the four-quarter moving average. (See column 3.) All the moving averages are still positioned between the quarters.

- For example, the first moving average (8.500) is positioned between spring and summer of 2000.

Step 3 The moving averages are then centered. The first centered moving average is found by (8.500+8.450)/2=8.475 and centered opposite summer 2000.

- The second moving average is found by (8.450+8.450)/2=8.45. The others are found similarly.

Step 4 The **specific seasonal** for each quarter is then computed by dividing the sales in column 1 by the centered moving average in column 4. The specific seasonal reports the ratio of the original time series value to the moving average.

- To explain further, if the time series is represented by *TSCI* and the moving average by *TCI,* then, algebraically, if we compute *TSCI*/*TCI,* the result is the seasonal component.
- The specific seasonal for the summer quarter of 2000 is 1.180, found by 10/8.475.

Step 5 The specific seasonals are organized in the following Table (Table 4). This table will help us locate the specific seasonals for the corresponding quarters.

- The values 1.180, 1.130, 1.141, 1.126, and 1.143 all represent estimates of the typical seasonal index for the summer quarter.
- A reasonable method to find a typical seasonal index is to average these values.
- So we find the typical index for the summer quarter by $(1.180+1.130+1.141+1.126+1.143)/5=1.144$.
- We used the arithmetic mean.

Table 4

Year	Winter	Spring	Summer	Fall	
2000			1.180	1.503	
2001	0.772	0.540	1.130	1.550	
2002	0.775	0.553	1.141	1.535	
2003	0.753	0.581	1.126	1.558	
2004	0.733	0.590	1.143	1.466	
2005	0.801	0.615			
Total	3.834	2.879	5.720	7.612	
Mean	0.767	0.576	1.144	1.522	4.009
Adjusted	0.765	0.575	1.141	1.519	4.000
Index	76.5	57.5	114.1	151.9	

Step 6 The four quarterly means (0.767, 0.576, 1.144, and 1.522) should theoretically total 4.00 because the average is set at 1.0.

- The total of the four quarterly means may not exactly equal 4.00 due to rounding.
- In this problem the total of the means is 4.009.
- A *correction factor* is therefore applied to each of the four means to force them to total 4.00.

CORRECTION FACTOR FOR ADJUSTING QUARTERLY MEANS

$$\text{Correction factor} = \frac{4.00}{\text{Total of four means}}$$

- In this example,

$$\text{Correction factor} = \frac{4.00}{4.009} = 0.997755$$

- The adjusted winter quarterly index is, therefore, .767 * .997755 = .765.
- Each of the means is adjusted downward so that the total of the four quarterly means is 4.00.
- Usually indexes are reported as percentages, so each value in the last row of the above Table (Table 4) has been multiplied by 100.

- So the index for the winter quarter is 76.5 and for the fall it is 151.9. How are these values interpreted?
- Sales for the fall quarter are 51.9 percent above the typical quarter, and for winter they are 23.5 below the typical quarter (100.0 - 76.5).

Calculation of Seasonal Indexes

	1	2	3	4	
1			1.180	1.503	
2	0.772	0.540	1.130	1.550	
3	0.775	0.553	1.141	1.535	
4	0.753	0.581	1.126	1.558	
5	0.733	0.590	1.143	1.466	
6	0.801	0.615			
mean:	0.767	0.576	1.144	1.522	4.009
adjusted:	0.765	0.575	1.141	1.519	4.000